

Physics

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On the Two Types of Charge Invariance

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The hypothesis of charge independence requires a very high degree of symmetry in the system of meson and nucleon fields; in particular it requires the invariance of the Hamiltonian of the system relative to the group of rotations in charge space. We know that this hypothesis has still not been definitely confirmed. However, a lower type of symmetry may be considered as established:¹ the invariance of the Hamiltonian relative to rotation around the z-axis in charge space (which corresponds to the law of conservation of the total charge system), and its invariance relative to rotation around the x-axis by 180° (which means invariance relative to the replacement of neutrons by protons and to the corresponding replacement of π^- -mesons by π^+ -mesons and vice versa). In other words, invariance relative to the group D_∞ in charge space has been established.

The proof of the invariance relative to the group of rotations must essentially include the neutral meson field. The interaction Hamiltonian of the meson and nucleon fields, which satisfies the requirement of invariance relative to the group of rotations, is a scalar in the isotopic space $g(\vec{\tau}, \vec{\varphi})$. A quite general type of interaction Hamiltonian satisfying the requirement of invariance relative to the D_∞ -type group can be written as

$$g(\tau_1\varphi_1 + \tau_2\varphi_2) + g_3\tau_3\varphi_3 + g_0\varphi_0,$$

where φ_3 is the third component of a vector in isotopic space and φ_0 is a scalar. If we disregard the possibility of describing the π^0 -mesons simultaneously by two functions φ_3 and φ_0 , we must determine whether the neutral meson field is described by the function φ_0 or by φ_3 .

The photoproduction of π^0 -mesons on deuterons is an effect which is very sensitive to the properties of the symmetry of the wave function of the neutral meson field in isotopic space:

$$\gamma + d \rightarrow \begin{cases} d + \pi^0 & \text{(I)} \\ n + p + \pi^0 & \text{(II)} \end{cases}$$

In an examination of this effect² it was found that when the signs of the interaction constants of the π^0 -mesons with the neutron g_n and the proton g_p are opposite, i.e., $g_n = -g_p$ (which corresponds to the description of the π^0 -mesons by the function φ_3), then the cross sections of processes (I) and (II) must be of the same order; when, on the other hand, $g_n = g_p$ (the π^0 -mesons are described by the function φ_0), the cross section of process (I) is much smaller than that of process (II). However, some assumptions were made in reference 2 which impair the generality of the results: the approximation of weak coupling, the adoption of a concrete meson theory, and the

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phenomenological introduction of anomalous magnetic moments, all of which play an important role. We shall show presently that this result can be obtained in general if we base ourselves only on the group properties of the wave functions and of the Hamiltonian in charge space as well as on one very natural assumption, which we shall explain a little later.

We shall write the Schroedinger equation for the system of the meson and nucleon field:

$$i \frac{\partial \Psi}{\partial t} = H_0 \Psi. \quad (1)$$

Here H_0 is invariant, at least relative to the D_∞ group and possibly relative to the group of rotations. In order to introduce the electromagnetic field into H_0 , we must replace all the $\partial/\partial x_i$ that act on the nucleon functions by

$$\frac{\partial}{\partial x_i} + ie \frac{(1 + \tau_3)}{2} A_i, \quad (2)$$

and all the $\partial/\partial X_i$ that act on the meson functions by

$$\frac{\partial}{\partial X_i} + ie L_3 A_i. \quad (3)$$

Here L_3 is the charge operator of the meson field which has the following characteristic values: +1 when it acts upon a π^+ -meson field, -1 when it acts upon a π^- -meson field, and 0 when it acts upon a π^0 -meson field. The operator has the transformation properties of a third component of a vector in isotopic space. We shall expand H in a power series of eA_i and limit ourselves to the first order (we shall consider only the absorption of the photon). In other words, the interaction with the electromagnetic field is assumed to be weak and viewed as a minor perturbation causing transitions in the system of the meson and nucleon fields. Then the Schroedinger equation takes the form

$$i \frac{\partial \Psi}{\partial t} = (H_0 + H_e) \Psi. \quad (4)$$

The matrix elements of the transition are

$$(\Psi_f, H_e \Psi_0), \quad (5)$$

where Ψ_f and Ψ_0 are functions satisfying Eq. (1).

If we use the property of symmetry of H_0 only with respect to a 180° rotation around the x -axis in isotopic space, then for both types of symmetry, H_e can be broken down into two terms:

$$H_e = S + V_3, \quad (6)$$

as can be seen from (2) and (3).

Here, the first term S , depending only on the interaction of the electromagnetic field with the charge of the nucleon, does not change with a 180° rotation, while the second term V_3 changes sign. For the case of charge independence, H_e in the form of (6) was obtained in reference 3 from special considerations (in that case, S and V_3 are a scalar and a third vector component, respectively). As can be seen from our derivation, Eq. (6) follows automatically from the general principles of quantum mechanics.

We shall examine process (I): Ψ_0 describes a deuteron, and Ψ_f a deuteron and a π^0 -meson. Since the wave function of the deuteron, antisymmetrical in

the proton and neutron coordinates, enters the matrix element twice (in Ψ_0 and Ψ_f), the sign of the product of the Ψ_0 and Ψ_f functions, under a 180° rotation around the x -axis, will be determined by the transformation properties of the meson function. If the meson function is a scalar (φ_0) in isotopic space, the product of the Ψ_0 and Ψ_f functions does not change sign under the rotation. If, on the other hand, the π^0 -mesons are described by the function φ_3 , the product of the Ψ_0 and Ψ_f functions will change sign.

We shall assume that the π^0 -mesons are described by the function φ_3 . Then, making a 180° rotation in isotopic space and noting the trivial assertion that the value of the matrix elements does not change when the integration variables are replaced, we can write instead of (5):

$$(\Psi_f, S + V_3 \Psi_0) = -(\Psi_f, S - V_3 \Psi_0) = (\Psi_f, V_3 \Psi_0). \quad (7)$$

If the π^0 -mesons are described by the function φ_0 , we have

$$(\Psi_f, S + V_3 \Psi_0) = (\Psi_f, S - V_3 \Psi_0) = (\Psi_f, S \Psi_0). \quad (8)$$

Hence, if we disregard the nucleon recoil effects, $S = 0$, the matrix element of transition for the process $\gamma + d \rightarrow d + \pi^0$ is zero when the π^0 -mesons are described by φ_0 .

The final wave function of two nucleons for reaction (II) may or may not change sign under the 180° rotation; therefore, for reaction (II) there are no selection rules that follow from the requirement of symmetry in isotopic space, except for the condition that, in the case of φ_0 , the final triplet state will predominate, while in the case of φ_3 the singlet isotopic state will predominate.

In the above discussion, we assumed that the anomalous magnetic moment of the nucleon (a. m. m.) is caused by π -mesons. Now, if we assume that the a. m. m. is not caused by π -mesons, it can be introduced phenomenologically. This will not change our reasoning. During the computation in the approximation of weak coupling, the a. m. m.'s introduced were equal to those measured in a static magnetic field. The anomalous parts of the magnetic moments of the neutron and proton are equal in magnitude and opposite in sign (with a sufficient accuracy for our purpose). Therefore, the part of H_e that was caused by the interaction with the a. m. m. had the form

$$\mu' (\tau_3^{(1)} + \tau_3^{(2)}),$$

i.e., it possessed the transformation properties of V_3 . Therefore, the a. m. m. made a large contribution to the cross section for $g_n = -g_p$ (or φ_3) and did not contribute at all when $g_n = g_p$ (or φ_0). Thus, the results obtained with perturbation theory follow directly from these general considerations. We note that if neutral mesons are described by φ_0 , their elastic photo-production on nuclei that are symmetrical with respect to neutrons and protons must be forbidden. The forbidden processes about which we have been speaking must be understood in the sense that they are forbidden only with an accuracy up to the terms describing the effect of nucleon recoil. We assume that

$$S \sim \frac{\mu}{M} V_3, \quad (9)$$

and, consequently, the cross sections of the forbidden processes are $(\mu/M)^2 \sim 40$ times smaller than the cross sections of the allowed processes. The natural assumption (9) is fundamental for our results. In this connection,

we computed the cross sections of processes (I) and (II) according to the pseudoscalar theory, taking radiation damping into consideration, and using the impulse approximation. The theory of radiation damping apparently gives an approximation closer to the true solution of Eq. (1) than the usual perturbation theory. In particular, this theory makes it possible, as we have shown, to find the magnitude and the angle and energy dependences of the cross sections for photoproduction of π^0 -mesons on nucleons without even the phenomenological introduction of the a. m. m. The computations of photoproduction in deuterium, in full agreement with the general considerations formulated in (7), (8), and (9), have shown that the cross sections of processes (I) and (II) are of the same order of magnitude when the π^0 -meson field is described by the function φ_3 , and that the cross section of process (I) is much smaller than the cross section of process (II) when the π^0 -mesons are described by the function φ_0 . Here, the results of the computations change very little with the phenomenological introduction of the a. m. m.

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¹Anderson, Fermi, Nagle, and Yodh, Phys. Rev., 86, 413 (1952); H. W. Wilson and W. H. Barkas, Phys. Rev., 89, 758 (1953).

²V. V. Mikhailov and A. M. Baldin, Doklady Akad. Nauk SSSR, 84, 47 (1952).

³K. M. Watson, Phys. Rev., 85, 852 (1952).

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